C2) Integrals - Intro to differential equations.

Quick derivative recap:

Δ = "change in..."
Δx = dx , the derivative of x

In order to calculate the slope of a graph (assuming a function in terms of x and y), we find the ratio between the change in y (Δy) and the change in x (Δx). This ratio, the slope, is the derivative of a function at a given input. In physics, this allows us to find an particle's velocity, acceleration, and so on, from a function of its position.

Integration is the opposite of a derivative. It allows us to find the position of an particle from its velocity, or the velocity of an particle from its acceleration.

If we know dx, we can find the **displacement of x** (how far x has moved) over an interval.
If we know dy, we can find the **displacement of y** over an interval.

<u>In terms of functions, integration has two basic steps.</u>

1. Isolate dx and all x variable terms on one side of an equation, and dy and all y variable terms on the other.
2. Take the antiderivative.
    a. Add the constant of integration (+c)

"+c" accounts for the starting position of the particle, without it you can only find displacement.

It may be helpful to think of integrating geometrically, in fact, this is often used to estimate integrals.

While taking the derivative finds the slope of a tangent line, the integral finds the area between the curve and the axis over an interval.

For example, $\int_a^b x^n dx$ is definite integral representing the area between x^n and the x axis over the interval from a to b.

Definite integral: an integral that has limits, and will give you a value (displacement)
Indefinite integral: an integral that does not have limits, and will give you a function+c

Solving definite integrals by hand:

1. Find the antiderivative of the function
2. Find the difference between the value of the antiderivative function at the beginning and end of the interval: [f(b) - f(a)] for $\int_a^b x^n dx$
3. Add the starting condition (position at time a)

Example:

$$\int_1^2 (x^2 + x^{-2})dx \;=\; \left[\; \tfrac{1}{3}x^3 - \tfrac{1}{x} \right]_1^2 = (\tfrac{1}{3}(2)^3 - \tfrac{1}{2}) - (\tfrac{1}{3}(1)^3 - \tfrac{1}{1}) \;=\; \tfrac{17}{6}$$

Paul Dawkins wrote this example problem, and his website may also be helpful to you. However, his notation is sometimes incorrect, so watch out for missing parentheses, etc.

In physics, integration is often performed in terms of x and t, which shows the motion of a particle with respect to time.

$\frac{dx}{dt}$ = velocity

Thus, we can find the position of that particle at any time using integration.

It is helpful to memorize basic integration rules, just like the basic derivative rules.

There are many methods to integrate functions that you cannot simply differentiate backwards, but here are some basics:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$
$$\int e^x dx = e^x + c$$
$$\int \frac{1}{x}\, dx = ln\,|x| + c$$
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Additional antiderivative rules, such as trig rules, can be found online.